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Title: Relaxations of AC Minimal Load-Shedding for Severe Contingency Analysis

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Relaxations of AC Minimal Load-Shedding for Severe Contingency Analysis

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Los Alamos National Laboratory
Advanced Network Science Initiative

Context

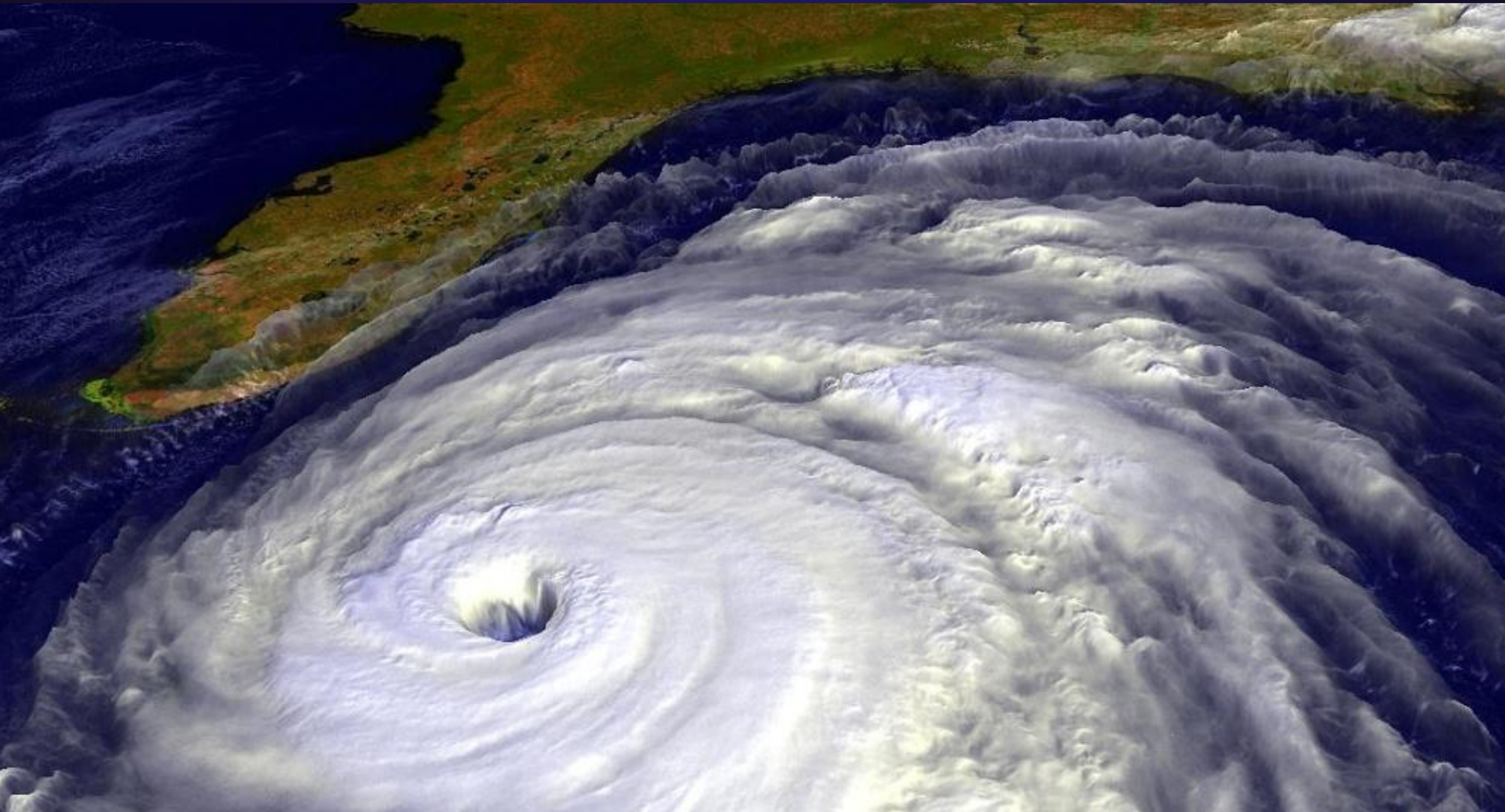
Los Alamos
NATIONAL LABORATORY
EST. 1943

DEPARTMENT OF ENERGY · UNITED STATES OF AMERICA

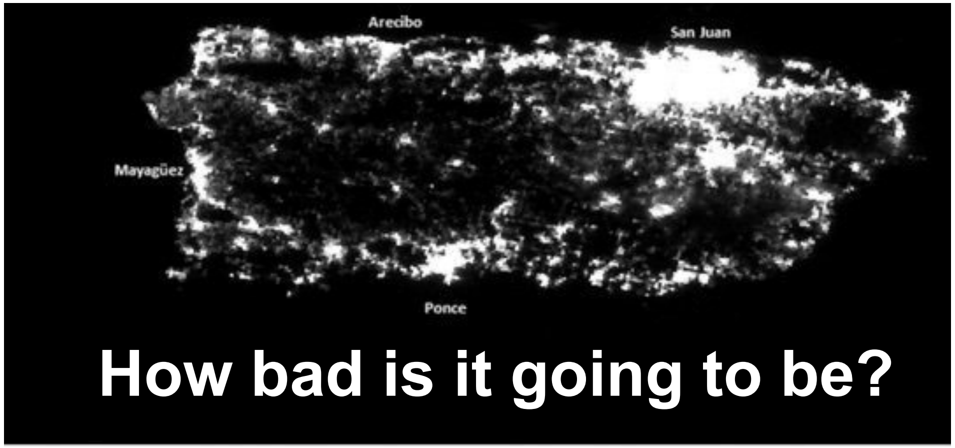
U.S. DEPARTMENT OF HOMELAND SECURITY

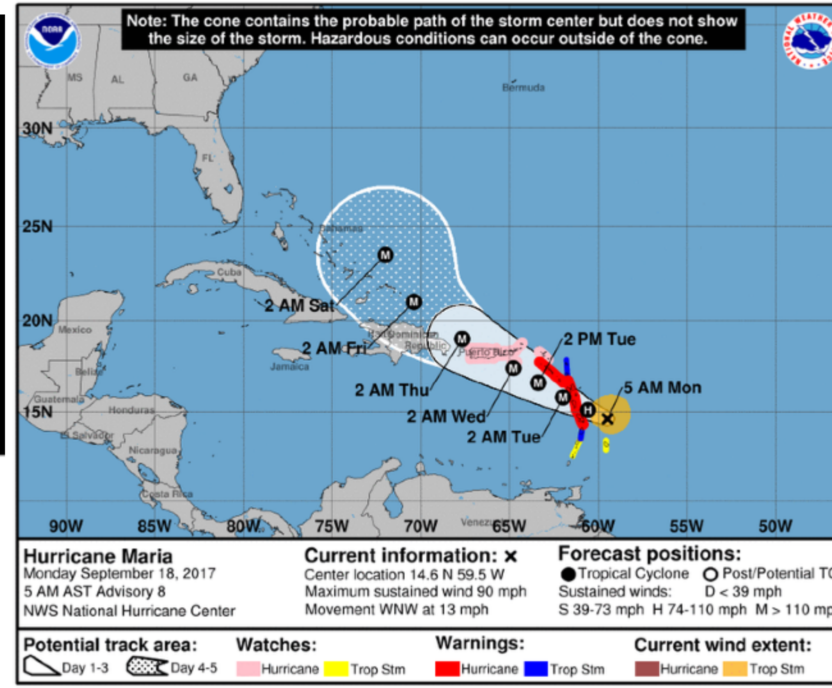
Motivation

Natural Disasters (hurricane Floyd ’11)

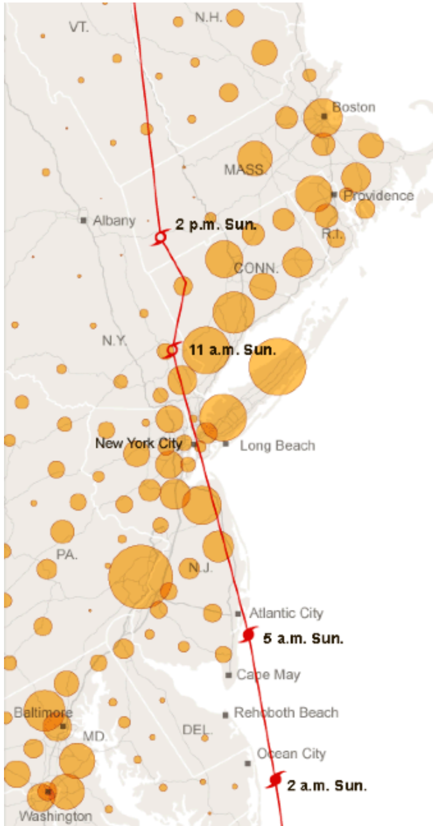


Natural Disasters (hurricane Maria ’17)





Natural Disasters (Irma ’11, Sandy ’12)



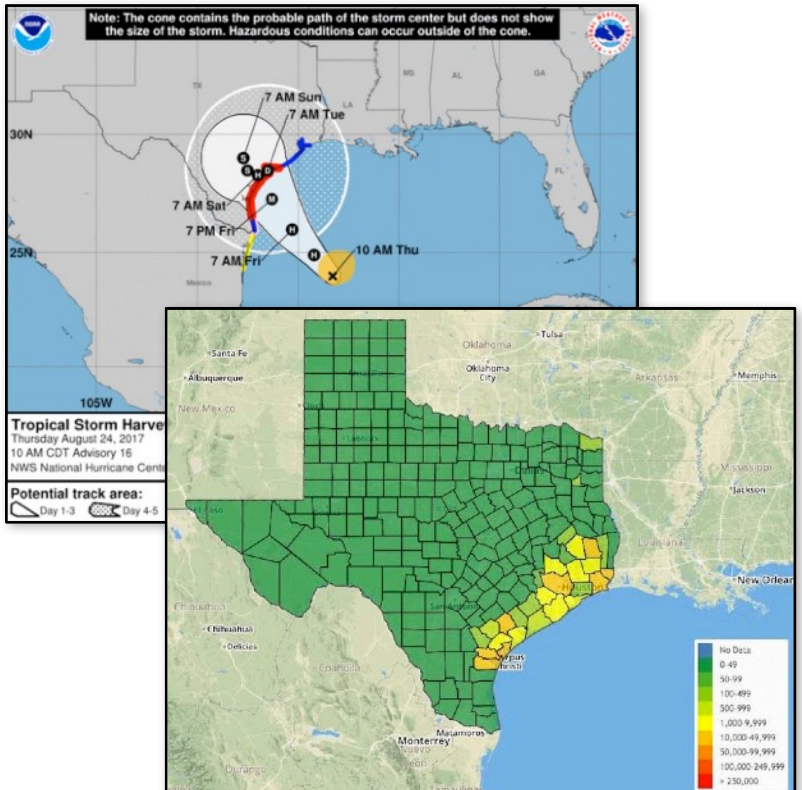
Large Scale

Very Costly

Targeted Attacks (e.g. metcalf sniper attack)

Inquiring Minds What to Know

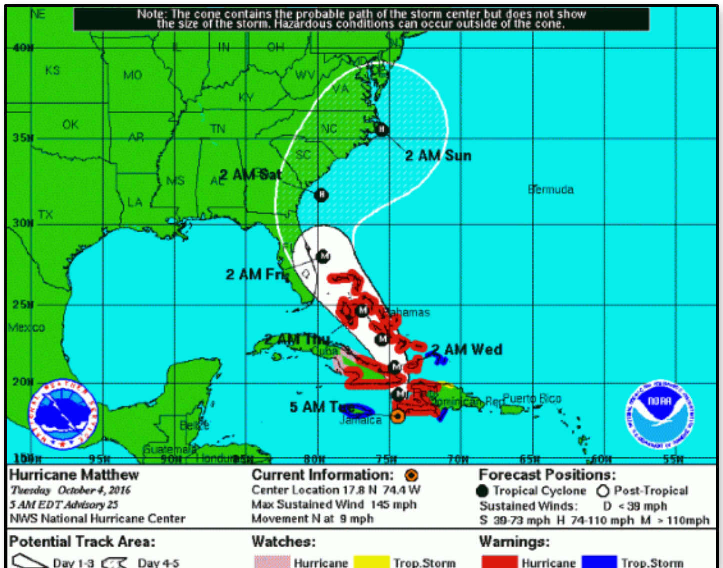
Understand Vulnerability



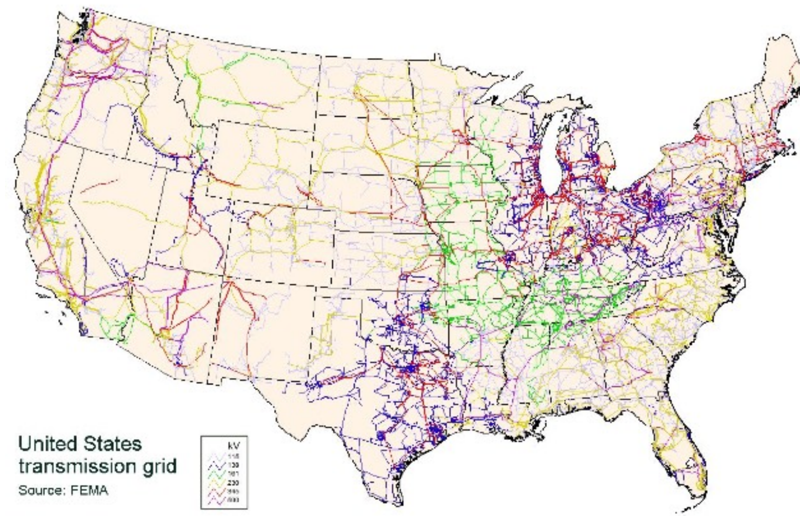
Mitigation

Challenges

Some Things We Know

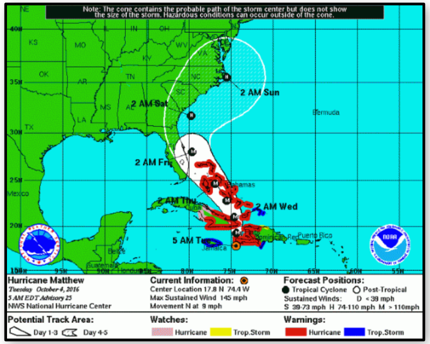


NOAA Storm Tracks



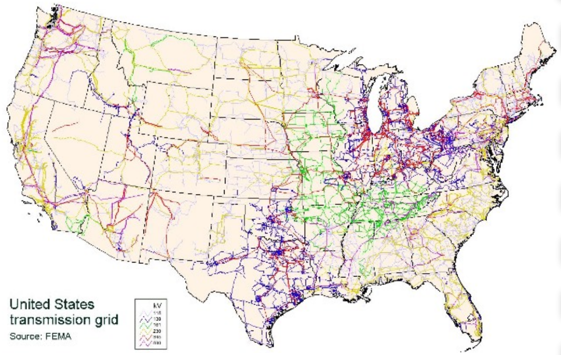
FERC 715 Filings (AC Transmission Systems)

The simplest thing I can think of…



Simulate Hurricanes

Estimate Damage



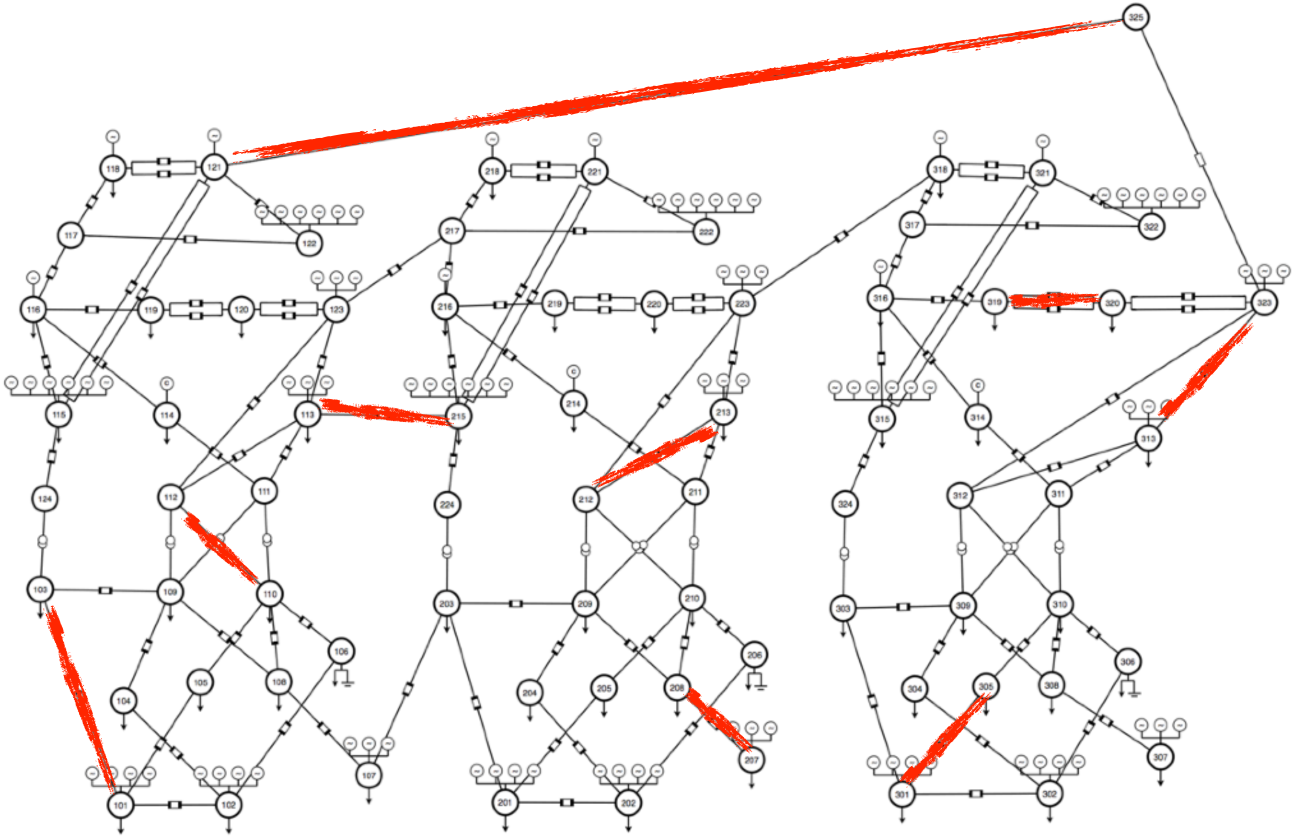
Solve AC Power Flow

SIEMENS

MATPOWER

PowerWorld Corporation

Applying Damage



http://immersive.erc.monash.edu.au/stac/

An Inconvenient Fact

NO SOLUTION

MATPOWER

PowerWorld Corporation

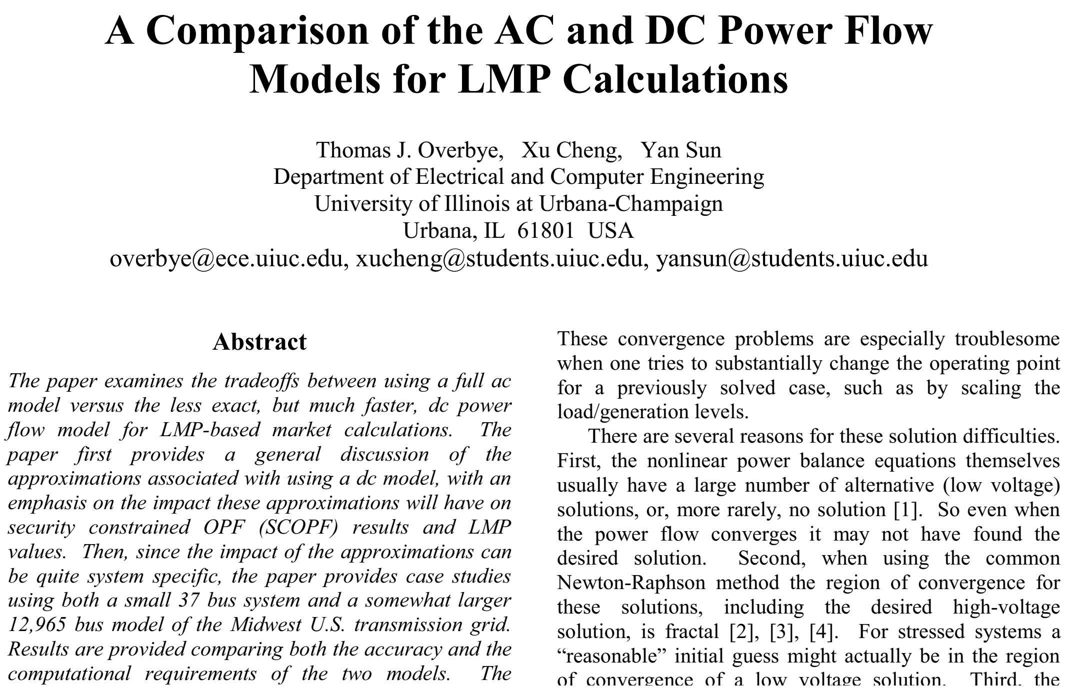
SIEMENS

By Hand:
Shed Loads,
Re-dispatch Generators

Very Difficult!

AC Power Flow Solver Challenges

- Finding a solution to the AC power flow equations, without a base-point solution, is "maddeningly difficult"



A Comparison of the AC and DC Power Flow Models for LMP Calculations

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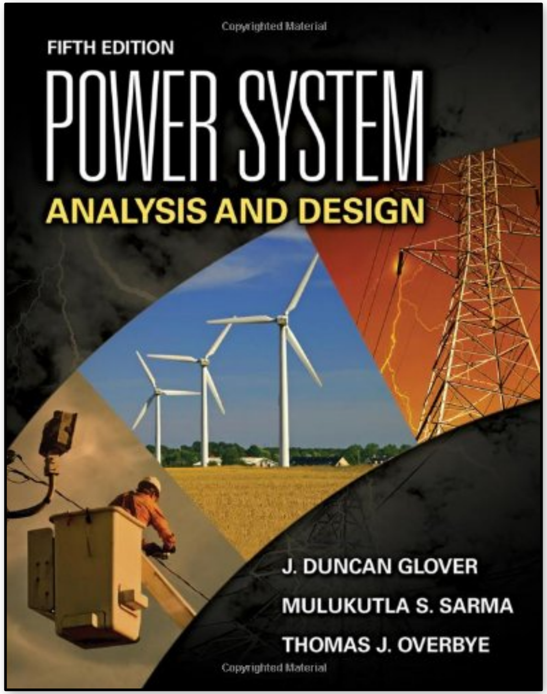
Abstract

The paper examines the tradeoffs between using a full ac model versus the less exact, but much faster, dc power flow model for LMP-based market calculations. The paper first provides a general discussion of the approximations associated with using a dc model, with an emphasis on the impact these approximations will have on security constrained OPF (SCOPF) results and LMP values. Then, since the impact of the approximations can be quite system specific, the paper provides case studies using both a small 37 bus system and a somewhat larger 12,965 bus model of the Midwest U.S. transmission grid. Results are provided comparing both the accuracy and the computational requirements of the two models. The

These convergence problems are especially troublesome when one tries to substantially change the operating point for a previously solved case, such as by scaling the load/generation levels.

There are several reasons for these solution difficulties. First, the nonlinear power balance equations themselves usually have a large number of alternative (low voltage) solutions, or, more rarely, no solution [1]. So even when the power flow converges it may not have found the desired solution. Second, when using the common Newton-Raphson method the region of convergence for these solutions, including the desired high-voltage solution, is fractal [2], [3], [4]. For stressed systems a "reasonable" initial guess might actually be in the region of convergence of a low voltage solution. Third, the



Some AC Power Flow "Optimization Tricks"

A Linear-Programming Approximation of AC Power Flows

Carleton Coffrin, *Member, IEEE*, Pascal Van Hentenryck, *Member, IEEE*

Accurate Load and Generation Scheduling for Linearized DC Models with Contingencies

Carleton Coffrin, *Student Member, IEEE*, Pascal Van Hentenryck, *Member, IEEE*, and Russell Bent

Transmission system restoration with co-optimization of repairs, load pickups, and generation dispatch ☆

CrossMark

Carleton Coffrin [a,b,c,*], Pascal Van Hentenryck [a,b]

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ABSTRACT

This paper studies the restoration of a transmission system after a significant disruption such as a natural disaster. It considers the co-optimization of repairs, load pickups, and generation dispatch to produce a sequencing of the repairs that minimizes the size of the blackout over time. The core of this process is a Restoration Ordering Problem (ROP), a non-convex mixed-integer nonlinear program that is outside the capabilities of existing solver technologies. To address this computational barrier, the paper examines two approximations of the power flow equations: The DC model and the recently proposed LPAC model. Systematic, large-scale testing indicates that the DC model is not sufficiently accurate for solving the ROP. In contrast, the LPAC power flow model, which captures line losses, reactive power, and voltage magnitudes, is sufficiently accurate to obtain restoration plans that can be converted into AC-feasible power flows. An experimental study also suggests that the LPAC model provides a robust and appealing tradeoff between accuracy and computational performance for solving the ROP.

Introduction

Restoring a power system after a significant disruption (e.g., a natural disaster) is an important task with consequences on both human and economic welfare. To mitigate the consequences of such events, the next generation of power system is expected to be more resilient and *self healing* [1]. This work focuses on the restoration of the transmission system, which is computationally challenging for a variety of reasons. First, since no *typical* operating point is known for the damaged network, it is often difficult to determine a steady-state power flow for the network, i.e., a solution to the AC power flow problem [2]. Second, restoration plans must jointly optimize the routing of repair crews, the scheduling of component energizing, load pickups, and generation dispatch. The resulting optimization problem is a non-convex mixed-integer nonlinear program, which is extremely hard from a computational standpoint.

These difficulties are addressed in the power restoration algorithm proposed in [3] by decomposing the problem into several steps. The algorithm decouples the power system and logistics aspects, first scheduling the component energizings and then routing the repair crews. The two subproblems are linked through precedence constraints that are derived from the power schedule and injected into the crew routing. This paper focuses on the first step of this decomposition, the so-called Restoration Order Problem (ROP). The goal of the ROP is to find a high-quality restoration schedule, i.e., a sequence of steady-state power flows for the network that minimizes the size of the blackout over time. Each steady-state corresponds to a restoration action (e.g., repairing a line) and may increase the served load and change the generation dispatch compared to earlier steady-states.

To find a high-quality restoration plan, the ROP formulation in [3] relies on the DC power flow approximation, which is widely used in power system optimization (e.g., [2,4–6]). However, the accuracy of the DC power flow is a topic of much discussion: Some papers take an favorable outlook, (e.g., [2,7]), while others (e.g., [8–11]) are more cautious. The accuracy of the DC power flow is particularly important in power restoration as a feasible AC base-point solution is often not available and it is preferable to operate the network near its design limits.

This paper investigates the usefulness of the DC power flow model and the recent LPAC power flow model [12] for optimization

☆ An earlier version of this work appeared in the Proceedings of the 18th Power Systems Computation Conference (PSCC), Wroclaw, Poland, August 2014.

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E-mail addresses: carleton.coffrin@nicta.com.au (C. Coffrin), pvh@nicta.com.au (P. Van Hentenryck).

Simple approximations, like DC Power Flow are not reliable

Done before convex relaxations were well understood (by me at least)

Goal of this work:

Go beyond Power Flow approximations.
Explore if Convex Relaxations and Non-Linear Programming can be used to solve this problem on realistic datasets (i.e. FERC 715).

Overview

- Motivation
- AC Optimal Power Flow Problem (AC-OPF)
- Adapt AC-OPF to the AC with Damage Problem (Take 1)
- Preliminary Testing
- Revised Formulation of the AC with Damage Problem (Take 2)
- Experimental Evaluation

AC Optimal Power Flow Model

Some Preliminaries

- Accept Matpower AC mathematics
  - Mathematics captures 90% of real-world data

MATPOWER

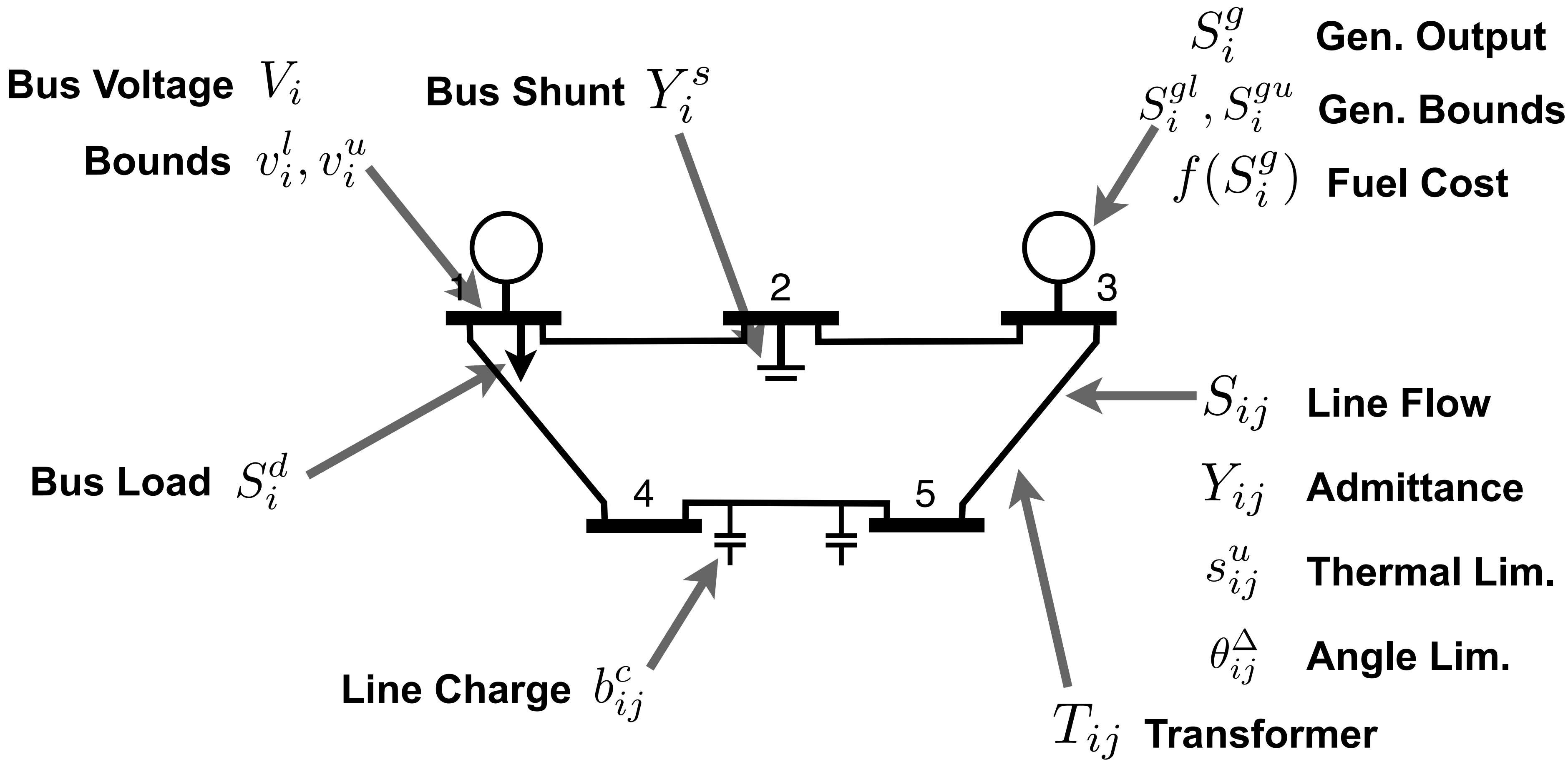
- Everything presented in complex numbers
  - Uppercase are complex numbers
  - Lowercase are real numbers
  - Bold values are constants

$$X = x + \boldsymbol{i}y$$

$$X^* = x - \boldsymbol{i}y$$

$$XX^* = |X|^2 = x^2 + y^2$$

Network Components and Parameters



Simple Power Flow Equations

Ohm’s Law on Lines

$$S_{ij} = \boldsymbol{Y}_{ij}^{*} V_i V_i^{*} - \boldsymbol{Y}_{ij}^{*} V_i V_j^{*}$$

Kirchhoff's Current Law (KCL) on buses

$$S_i^g - \boldsymbol{S}_i^d = \sum_{(i,j) \in E \cup E^R} S_{ij}$$

Complete Power Flow Equations

Ohm's Law on Lines

$$S_{ij} = \left(\boldsymbol{Y}_{ij}^{*} - \boldsymbol{i}\frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2} \right) \frac{|V_i|^2}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}_{ij}^{*} \frac{V_i V_j^{*}}{\boldsymbol{T}_{ij}}$$

$$S_{ji} = \left(\boldsymbol{Y}_{ij}^{*} - \boldsymbol{i}\frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2} \right) |V_j|^2 - \boldsymbol{Y}_{ij}^{*} \frac{V_i^{*} V_j}{\boldsymbol{T}_{ij}^{*}}$$

Kirchhoff's Current Law (KCL) on buses

$$S_i^g - \boldsymbol{S}_i^d - \boldsymbol{Y}_i^s |V_i|^2 = \sum_{(i,j) \in E \cup E^R} S_{ij}$$

AC Optimal Power Flow Model (AC-OPF)

variables: $S_i^g(\forall i \in N),\ V_i(\forall i \in N)$

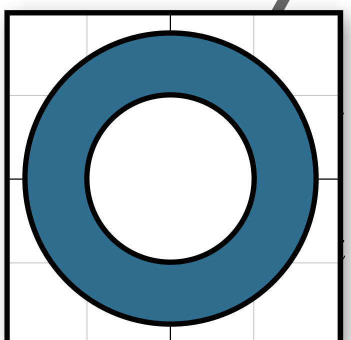
minimize: $\displaystyle\sum_{I \in N} \boldsymbol{c}_{2i}(\Re(S_i^g))^2 + \boldsymbol{c}_{1i}\Re(S_i^g) + \boldsymbol{c}_{0i}$ — **Fuel Cost Objective**

subject to:

$\boldsymbol{v}_i^{\boldsymbol{l}} \le |V_i| \le \boldsymbol{v}_i^{\boldsymbol{u}} \ \ \forall i \in N$ — **Voltage Bounds**

$\boldsymbol{S}_i^{\boldsymbol{gl}} \le S_i^g \le \boldsymbol{S}_i^{\boldsymbol{gu}} \ \ \forall i \in N$ — **Generation Bounds**

$S_i^g - \boldsymbol{S}_i^d - \boldsymbol{Y}_i^s|V_i|^2 = \displaystyle\sum_{(i,j)\in E \cup E^R} S_{ij} \ \ \forall i \in N$ — **KCL**



$= \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i}\dfrac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2}\right)\dfrac{|V_i|^2}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}_{ij}^*\dfrac{V_iV_j^*}{\boldsymbol{T}_{ij}} \ \ (i,j) \in E$

$= \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i}\dfrac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2}\right)|V_j|^2 - \boldsymbol{Y}_{ij}^*\dfrac{V_i^*V_j}{\boldsymbol{T}_{ij}^*} \ \ (i,j) \in E$

Ohm's Law

$|S_{ij}| \le \boldsymbol{s}_{ij}^{\boldsymbol{u}} \ \ \forall(i,j) \in E \cup E^R$ — **Thermal Limit**

$-\boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}} \le \angle(V_iV_j^*) \le \boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}} \ \ \forall(i,j) \in E$ — **Angle Limit**

NP-Hardness of AC-OPF

AC-Feasibility on Tree Networks is NP-Hard

Karsten Lehmann, Alban Grastien, and Pascal Van Hentenryck

***Abstract*—Recent years have witnessed significant interest in convex relaxations of the power flows, with several papers showing that the second-order cone relaxation is tight for tree networks under various conditions on loads or voltages. This paper shows that ac-feasibility, i.e., to find whether some generator dispatch can satisfy a given demand, is NP-hard for tree networks.**

***Index Terms*—Computational complexity, optimal power flow (OPF).**

Nomenclature

$\mathcal{N}$	AC-network.
N	Set of buses.
N_G	Set of generators.
N_L	Set of loads.
i	Bus of a network.
j	Bus of a network.
E	Set of lines.

I. Introduction

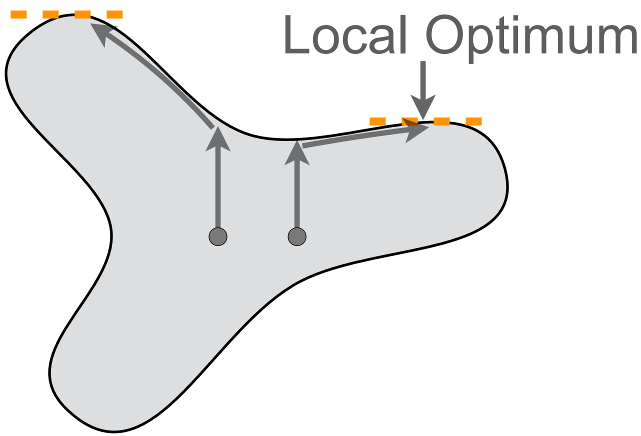
MANY interesting applications in power systems, including optimal power flows, optimize an objective function over the steady-state power flow equations, which are nonlinear and nonconvex. These applications typically include an *ac-feasibility* (AC-FEAS) subproblem: find whether some generator dispatch can satisfy a given demand.

Although the set of ac-feasible solutions is in general a nonconvex set, this does not imply that the ac-feasibility problem is NP-hard,[1] as nonconvexity does not imply NP-hardness. For example, the family of optimization problems $\min y$ such that $0 \leq y \leq \prod_{i=1}^{n} x_i$ where $n \in \mathbb{N}$ has a nonconvex constraint and a nonconvex solution set but the optimal solution is always $y = 0$ and can be trivially computed.

The first NP-hardness proof for ac-feasibility was given for a cyclic network structure in [1]. It relies on a variant of the dc model [2] but uses a sine function around the phase angle difference. From an ac perspective, this means that conductances

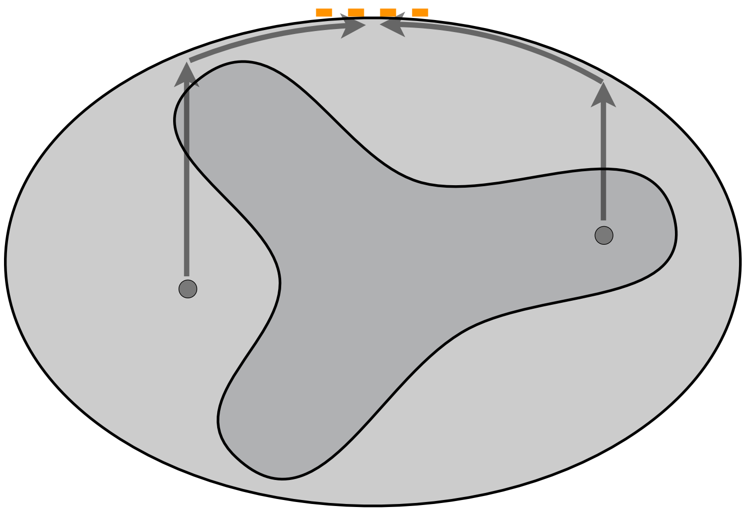
The Problem with Non-Convexity

Non-Convex
Interior Point Methods



Pro:
Can produce a feasible solution
Con:
No guarantees (opt. or feasible)

Convex Relaxation
Interior Point Methods



Pro:
Guarantees (opt!)
Con:
Can cheat (not non-convex feasible)

AC Optimal Power Flow Model (AC-OPF)

variables: $S_i^g(\forall i \in N),\ V_i(\forall i \in N)$

minimize: $$\sum_{I \in N} \boldsymbol{c}_{2i}(\Re(S_i^g))^2 + \boldsymbol{c}_{1i}\Re(S_i^g) + \boldsymbol{c}_{0i}$$

subject to:

$$\boldsymbol{v}_i^{\boldsymbol{l}} \le |V_i| \le \boldsymbol{v}_i^{\boldsymbol{u}} \quad \forall i \in N$$

$$\boldsymbol{S}_i^{\boldsymbol{gl}} \le S_i^g \le \boldsymbol{S}_i^{\boldsymbol{gu}} \quad \forall i \in N$$

$$S_i^g - \boldsymbol{S}_i^d - \boldsymbol{Y}_i^s |V_i|^2 = \sum_{(i,j)\in E \cup E^R} S_{ij} \quad \forall i \in N$$

$$S_{ij} = \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i}\frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2}\right)\frac{|V_i|^2}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}_{ij}^*\frac{V_i V_j^*}{\boldsymbol{T}_{ij}} \quad (i,j) \in E$$

$$S_{ji} = \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i}\frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2}\right)|V_j|^2 - \boldsymbol{Y}_{ij}^*\frac{V_i^* V_j}{\boldsymbol{T}_{ij}^*} \quad (i,j) \in E$$

$$|S_{ij}| \le \boldsymbol{s}_{ij}^{\boldsymbol{u}} \quad \forall (i,j) \in E \cup E^R$$

$$-\boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}} \le \angle(V_i V_j^*) \le \boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}} \quad \forall (i,j) \in E$$

How to convexify this thing?

$$V_i V_j^*$$

A Simple Second Order Cone Relaxation (SOC)

Radial Distribution Load Flow Using Conic Programming

Rabih A. Jabr, *Member, IEEE*

***Abstract*—This paper shows that the load flow problem of a radial distribution system can be modeled as a convex optimization problem, particularly a conic program. The implications of the conic programming formulation are threefold. First, the solution of the distribution load flow problem can be obtained in polynomial time using interior-point methods. Second, numerical ill-conditioning can be automatically alleviated by the use of scaling in the interior-point algorithm. Third, the conic formulation facilitates the inclusion of the distribution power flow equations in radial system optimization problems. A state-of-the-art implementation of an interior-point method for conic programming is used to obtain the solution of nine different distribution systems. Comparisons are carried out with a previously published radial load flow program by R. Cespedes.**

***Index Terms*—Load flow control, nonlinear programming, optimization methods.**

I. Introduction

THE LOAD flow program is an essential tool for the efficient operation and control of power distribution networks. The distribution systems are characterized by their prevailing radial nature and high R/X ratio. This renders the

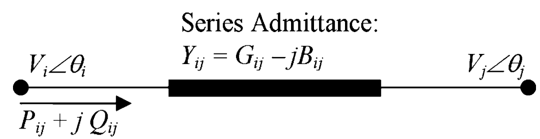


Fig. 1. Distribution line model.

connections is sufficient to describe fully the concepts in this paper.

The real/reactive power flows from node i to node j are

$$P_{ij} = G_{ij}V_i^2 - G_{ij}V_iV_j\cos\theta_{ij} + B_{ij}V_iV_j\sin\theta_{ij}, \quad (1)$$
$$Q_{ij} = B_{ij}V_i^2 - B_{ij}V_iV_j\cos\theta_{ij} - G_{ij}V_iV_j\sin\theta_{ij}, \quad (2)$$

where $\theta_{ij} = \theta_i - \theta_j$. By defining $u_i = V_i^2/\sqrt{2}$, $R_{ij} = V_iV_j\cos\theta_{ij}$, and $I_{ij} = V_iV_j\sin\theta_{ij}$, (1) and (2) become

$$P_{ij} = \sqrt{2}G_{ij}u_i - G_{ij}R_{ij} + B_{ij}I_{ij}, \quad (3)$$
$$Q_{ij} = \sqrt{2}B_{ij}u_i - B_{ij}R_{ij} - G_{ij}I_{ij}. \quad (4)$$

In (3) and (4), R_{ij} and I_{ij} are constrained such that

SOC Optimal Power Flow Model (SOC-OPF)

$$
\begin{aligned}
\textbf{variables: } & S^g_i (\forall i \in N),\ W_{ij} (\forall (i,j) \in E),\ W_{ii} (\forall i \in N) \\
\textbf{minimize: } & \sum_{i \in N} \boldsymbol{c}_{2i} (\Re(S^g_i))^2 + \boldsymbol{c}_{1i} \Re(S^g_i) + \boldsymbol{c}_{0i} \\
\textbf{subject to: } & \boldsymbol{v}^{\boldsymbol{l}}_i \le W_{ii} \le \boldsymbol{v}^{\boldsymbol{u}}_i \quad \forall i \in N \\
& \boldsymbol{S}^{\boldsymbol{gl}}_i \le S^g_i \le \boldsymbol{S}^{\boldsymbol{gu}}_i \quad \forall i \in N \\
& S^g_i - \boldsymbol{S}^d_i - \boldsymbol{Y}^s_i W_{ii} = \sum_{(i,j) \in E \cup E^R} S_{ij} \quad \forall i \in N \\
& S_{ij} = \left(\boldsymbol{Y}^*_{ij} - \boldsymbol{i} \frac{\boldsymbol{b}^{\boldsymbol{c}}_{ij}}{2} \right) \frac{W_{ii}}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}^*_{ij} \frac{W_{ij}}{\boldsymbol{T}_{ij}} \quad (i,j) \in E \\
& S_{ji} = \left(\boldsymbol{Y}^*_{ij} - \boldsymbol{i} \frac{\boldsymbol{b}^{\boldsymbol{c}}_{ij}}{2} \right) W_{jj} - \boldsymbol{Y}^*_{ij} \frac{W^*_{ij}}{\boldsymbol{T}^*_{ij}} \quad (i,j) \in E \\
& |S_{ij}| \le \boldsymbol{s}^{\boldsymbol{u}}_{ij} \quad \forall (i,j) \in E \cup E^R \\
& \tan(-\boldsymbol{\theta}^{\boldsymbol{\Delta}}_{ij}) \Re(W_{ij}) \le \Im(W_{ij}) \le \tan(\boldsymbol{\theta}^{\boldsymbol{\Delta}}_{ij}) \Re(W_{ij}) \quad \forall (i,j) \in E \\
& |W_{ij}|^2 \le W_{ii} W_{jj} \quad \forall (i,j) \in E
\end{aligned}
$$

$$W_{ij} = V_i V_j^*$$

Variable Lifting

$$|W_{ij}|^2 \le W_{ii} W_{jj}$$

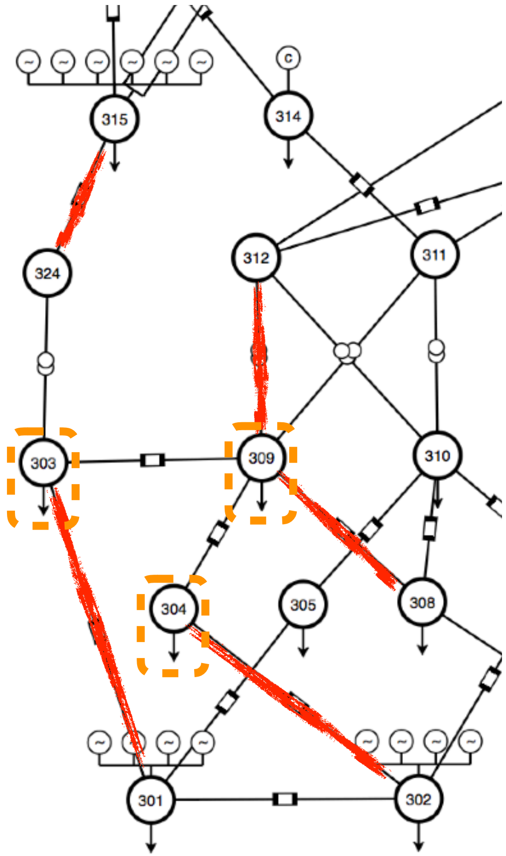
Valid Inequality

Valid Inequality

https://arxiv.org/abs/1502.07847

Adapting AC Optimal Power Flow for Component Damage

Adapting the AC-OPF to support Component Damage



$$\cancel{S_i^g} - \boldsymbol{S}_i^d - \cancel{\boldsymbol{Y}_i^s |V_i|^2} = \sum_{(i,j)\in E\cup E^R} \cancel{S_{ij}} \quad \forall i \in N$$

$$\boldsymbol{S}_i^d = 0 + \boldsymbol{i}0$$

Adapting the AC-OPF to support Component Damage

$$
\begin{aligned}
\textbf{variables:}\quad & S_i^g (\forall i \in N),\; V_i (\forall i \in N) \\
\textbf{minimize:}\quad & \sum_{I \in N} \boldsymbol{c}_{2i} (\Re(S_i^g))^2 + \boldsymbol{c}_{1i} \Re(S_i^g) + \boldsymbol{c}_{0i} \\
\textbf{subject to:}\quad & \boldsymbol{v}_i^{\boldsymbol{l}} \leq |V_i| \leq \boldsymbol{v}_i^{\boldsymbol{u}} \;\; \forall i \in N \\
& \boldsymbol{S}_i^{\boldsymbol{gl}} \leq S_i^g \leq \boldsymbol{S}_i^{\boldsymbol{gu}} \;\; \forall i \in N \\
& S_i^g - \boxed{\boldsymbol{S}_i^d} - \boldsymbol{Y}_i^s |V_i|^2 = \sum_{(i,j) \in E \cup E^R} S_{ij} \;\; \forall i \in N \\
& S_{ij} = \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i} \frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2} \right) \frac{|V_i|^2}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}_{ij}^* \frac{V_i V_j^*}{\boldsymbol{T}_{ij}} \;\; (i,j) \in E \\
& S_{ji} = \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i} \frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2} \right) |V_j|^2 - \boldsymbol{Y}_{ij}^* \frac{V_i^* V_j}{\boldsymbol{T}_{ij}^*} \;\; (i,j) \in E \\
& |S_{ij}| \leq \boldsymbol{s}_{ij}^{\boldsymbol{u}} \;\; \forall (i,j) \in E \cup E^R \\
& -\boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}} \leq \angle (V_i V_j^*) \leq \boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}} \;\; \forall (i,j) \in E
\end{aligned}
$$

Intuition, need to be able to shed loads. Trivial solution when 0 load in the system

AC Minimum Load Shedding Model (AC-MLS)

variables: $S^g_i (\forall i \in N),\ V_i (\forall i \in N),\ \boxed{z^d_i \in (0,1) (\forall i \in N)}$

maximize: $\boxed{\sum_{i \in N} |\Re(\boldsymbol{S}^d_i)| z^d_i}$

subject to:

$$\boldsymbol{v}^{\boldsymbol{l}}_i \le |V_i| \le \boldsymbol{v}^{\boldsymbol{u}}_i \quad \forall i \in N$$

$$\boldsymbol{S}^{\boldsymbol{gl}}_i \le S^g_i \le \boldsymbol{S}^{\boldsymbol{gu}}_i \quad \forall i \in N$$

$$S^g_i - \boxed{z^d_i \boldsymbol{S}^d_i} - \boldsymbol{Y}^s_i |V_i|^2 = \sum_{(i,j) \in E \cup E^R} S_{ij} \quad \forall i \in N$$

$$S_{ij} = \left(\boldsymbol{Y}^*_{ij} - \boldsymbol{i} \frac{\boldsymbol{b}^{\boldsymbol{c}}_{ij}}{2} \right) \frac{|V_i|^2}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}^*_{ij} \frac{V_i V^*_j}{\boldsymbol{T}_{ij}} \quad (i,j) \in E$$

$$S_{ji} = \left(\boldsymbol{Y}^*_{ij} - \boldsymbol{i} \frac{\boldsymbol{b}^{\boldsymbol{c}}_{ij}}{2} \right) |V_j|^2 - \boldsymbol{Y}^*_{ij} \frac{V^*_i V_j}{\boldsymbol{T}^*_{ij}} \quad (i,j) \in E$$

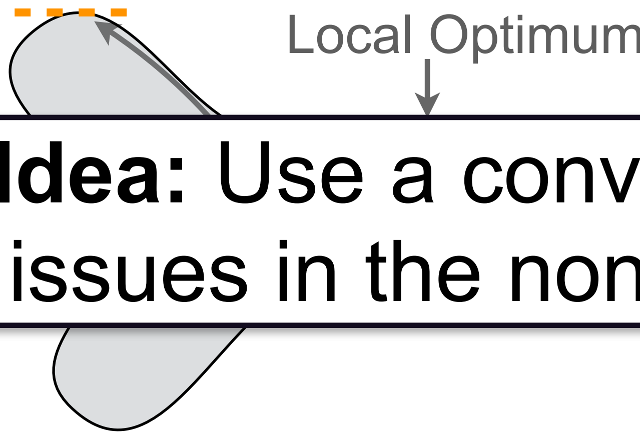
$$|S_{ij}| \le \boldsymbol{s}^{\boldsymbol{u}}_{ij} \quad \forall (i,j) \in E \cup E^R$$

$$-\boldsymbol{\theta}^{\boldsymbol{\Delta}}_{ij} \le \angle (V_i V^*_j) \le \boldsymbol{\theta}^{\boldsymbol{\Delta}}_{ij} \quad \forall (i,j) \in E$$

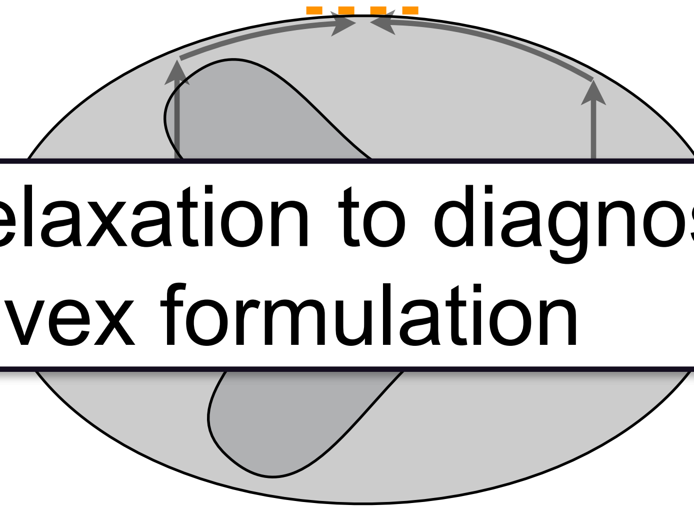
Is this a sufficient formulation?

The Problem with Non-Convexity

Non-Convex
Interior Point Methods



Convex
Interior Point Methods



Key Idea: Use a convex relaxation to diagnose issues in the non-convex formulation

Solver says Infeasible:
Algorithm / Formulation

Solver says Infeasible:
Formulation*

*up to floating point precision issues

AC Minimum Load Shedding Relaxation (SOC-MLS)

variables: $S_i^g (\forall i \in N),\ W_{ij}(\forall (i,j) \in E),\ W_{ii}(\forall i \in N),\ z_i^d \in (0,1)(\forall i \in N)$

maximize: $\boxed{\sum_{i \in N} |\Re(\boldsymbol{S}_i^d)| z_i^d}$

subject to:

$$(\boldsymbol{v}_i^{\boldsymbol{l}})^2 \leq W_{ii} \leq (\boldsymbol{v}_i^{\boldsymbol{u}})^2 \quad \forall i \in N$$

$$\boldsymbol{S}_i^{\boldsymbol{gl}} \leq S_i^g \leq \boldsymbol{S}_i^{\boldsymbol{gu}} \quad \forall i \in N$$

$$S_i^g - \boxed{z_i^d \boldsymbol{S}_i^d} - \boldsymbol{Y}_i^s W_{ii} = \sum_{(i,j) \in E \cup E^R} S_{ij} \quad \forall i \in N$$

$$S_{ij} = \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i}\frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2}\right)\frac{W_{ii}}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}_{ij}^*\frac{W_{ij}}{\boldsymbol{T}_{ij}} \quad (i,j) \in E$$

$$S_{ji} = \left(\boldsymbol{Y}_{ij}^* - \boldsymbol{i}\frac{\boldsymbol{b}_{ij}^{\boldsymbol{c}}}{2}\right) W_{jj} - \boldsymbol{Y}_{ij}^*\frac{W_{ij}^*}{\boldsymbol{T}_{ij}^*} \quad (i,j) \in E$$

$$|S_{ij}| \leq \boldsymbol{s}_{ij}^{\boldsymbol{u}} \quad \forall (i,j) \in E \cup E^R$$

$$\tan(-\boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}})\Re(W_{ij}) \leq \Im(W_{ij}) \leq \tan(\boldsymbol{\theta}_{ij}^{\boldsymbol{\Delta}})\Re(W_{ij}) \ \forall (i,j) \in E$$

$$|W_{ij}|^2 \leq W_{ii}W_{jj} \quad \forall (i,j) \in E$$

$$W_{ij} = V_i V_j^*$$

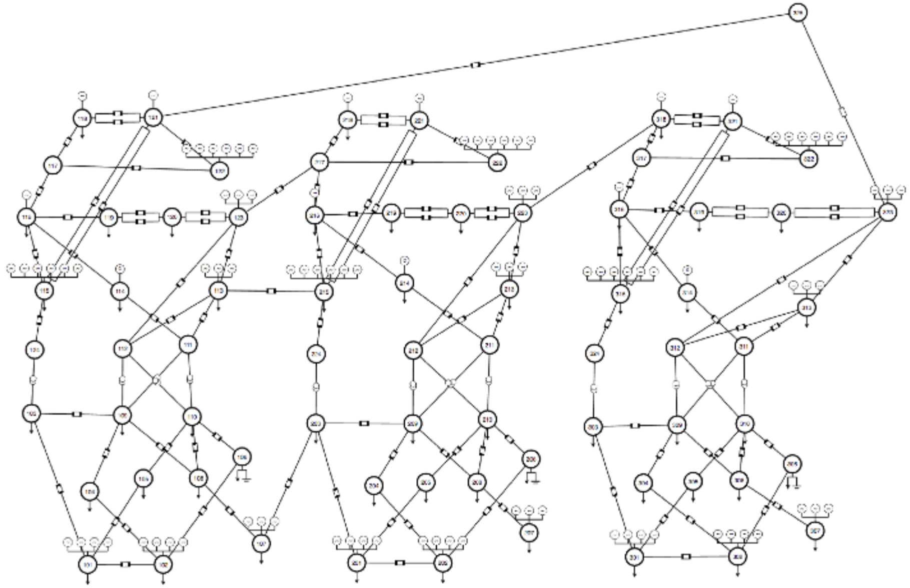
Variable Lifting

Valid Inequality

Testing the SOC-MLS Formulation

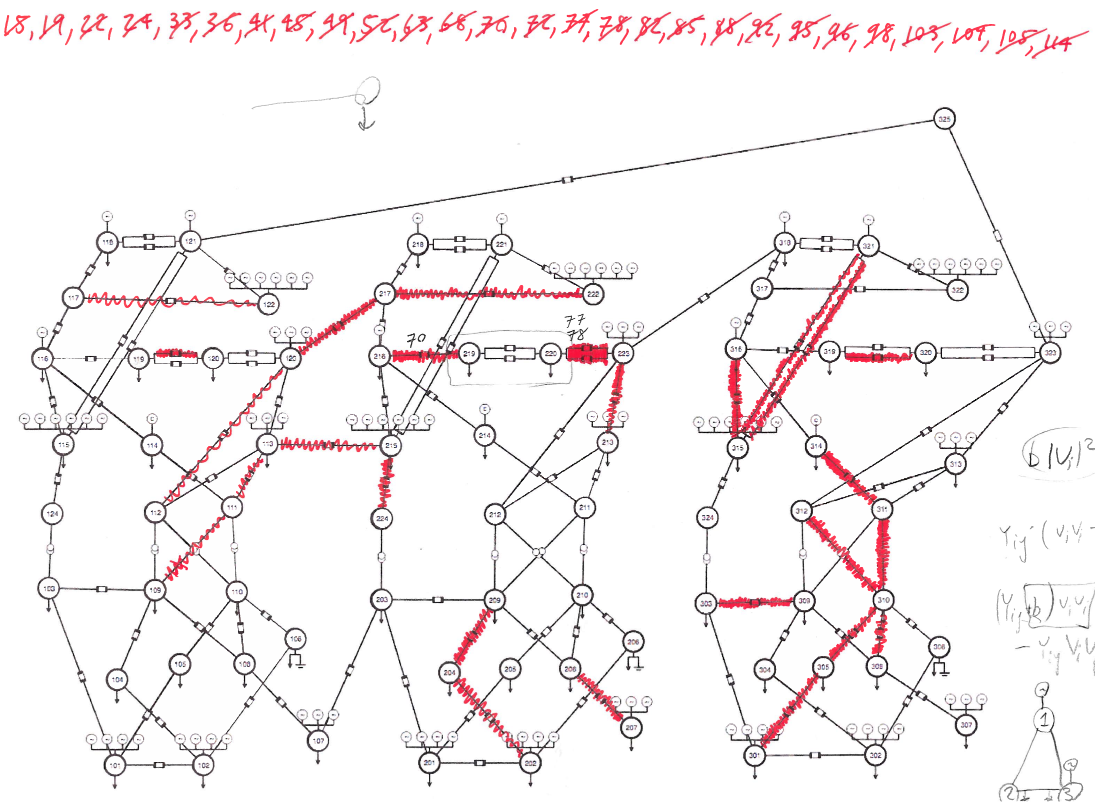
Testing the SOC-MLS Formulation

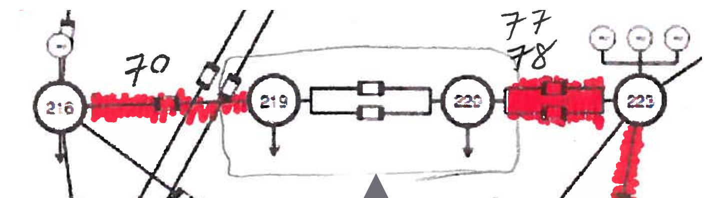
- Testing SOC-MLS on the IEEE RTS 96 network
  - A small but well curated test case
  - All N-1 Cases (bus, generator, branch)
  - 1000 random N-30% of branches cases
- Convergence on 99.0% of cases
  - Look at remaining 1.0% in detail
  - Counter examples by brute force



Testing the SOC-MLS Formulation

- **Example of an infeasible case**

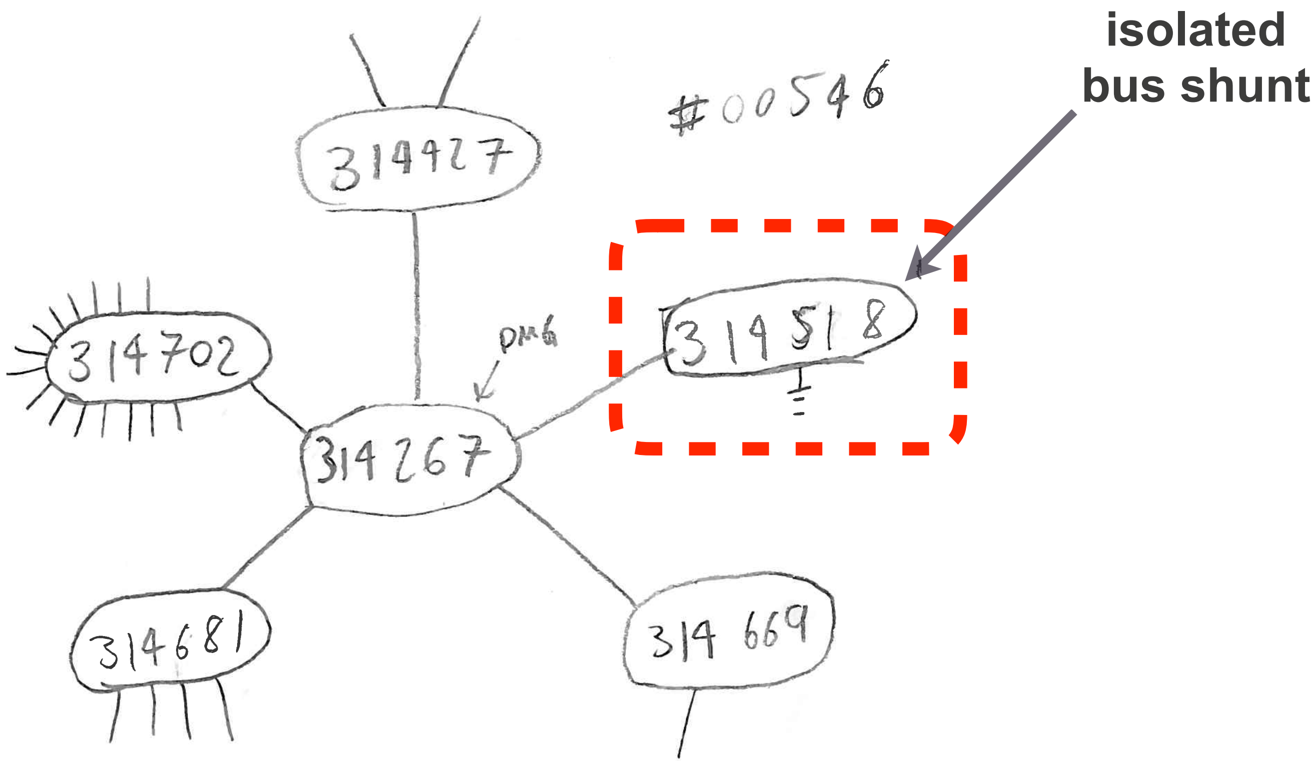




isolated line charge

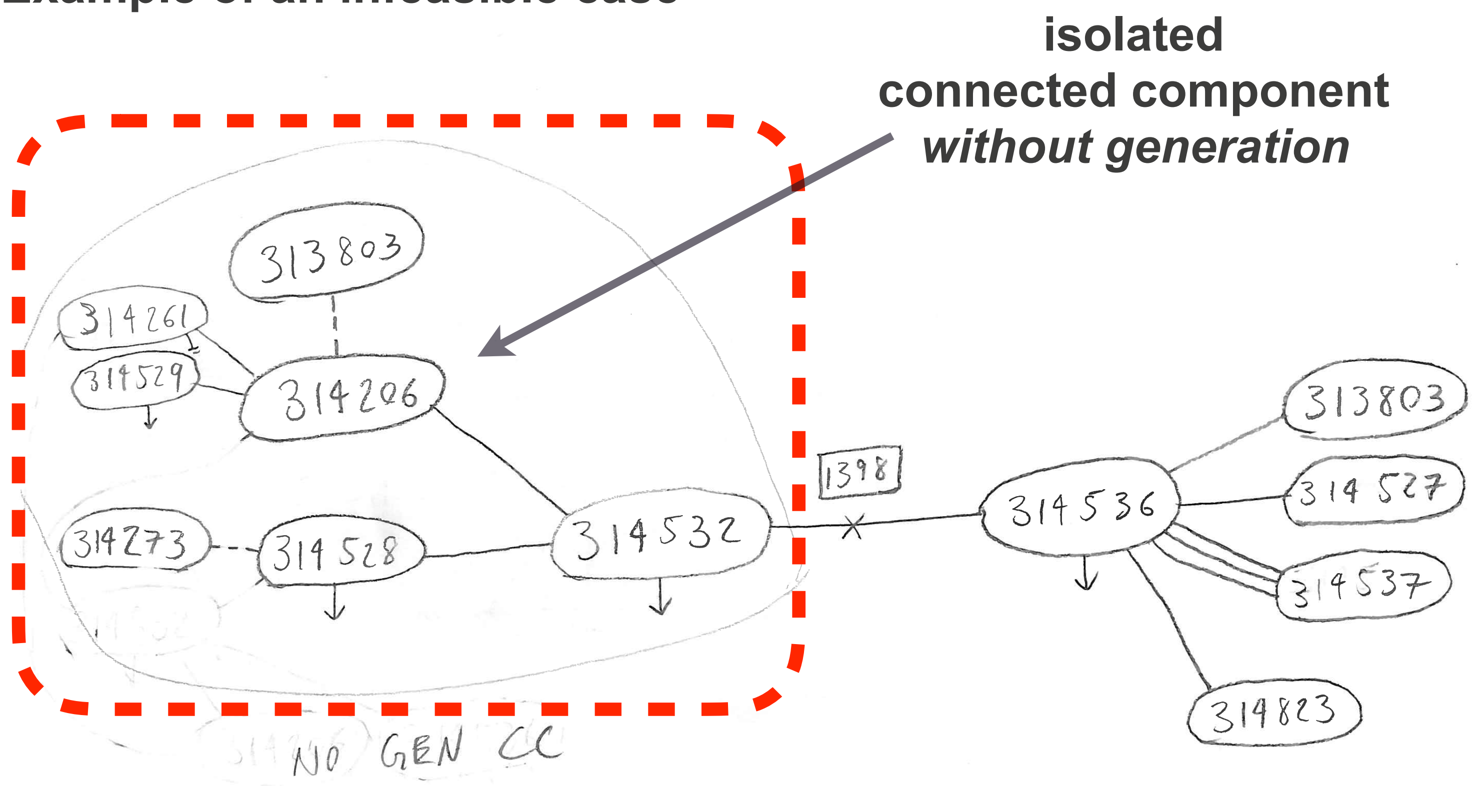
Testing the SOC-MLS Formulation

- **Example of an infeasible case**



Testing the SOC-MLS Formulation

- **Example of an infeasible case**



Testing the SOC-MLS Formulation

- Long-Story Short…

- The AC-MLS formulation requires at least 4 key features
  - Shed loads
  - Shed bus shunts
  - Un-commit Generators
  - Remove Buses

- AC-MLS preprocessing
  - Remove "dangling buses"
  - Remove "connected components without load or generation"
  - Solve one connected component at a time

Revised AC-MLS Formulation

Revised AC Minimum Load Shedding Model (AC-MLS)

- **4 Variants**
  - **AC-MLS (MINLP)**
  - **AC-MLS-C (NLP)**
  - **SOC-MLS (MISOCP)**
  - **SOC-MLS-C (SOCP)**

variables: $S^g_i (\forall i \in N),\ V_i (\forall i \in N)$

$$z^v_i, z^g_i \in \{0,1\} \ \ \forall i \in N$$

$$z^d_i, z^s_i \in (0,1) \ \ \forall i \in N$$

maximize: $\sum_{i \in N} z^v_i, \sum_{i \in N} z^g_i, \sum_{i \in N} z^s_i, \sum_{i \in N} |\Re(\boldsymbol{S}^d_i)| z^d_i$

subject to:

$$z^v_i \boldsymbol{v}^{\boldsymbol{l}}_i \le |V_i| \le z^v_i \boldsymbol{v}^{\boldsymbol{u}}_i \ \ \forall i \in N$$

$$z^g_i \boldsymbol{S}^{\boldsymbol{gl}}_i \le S^g_i \le z^g_i \boldsymbol{S}^{\boldsymbol{gu}}_i \ \ \forall i \in N$$

$$S^g_i - z^d_i \boldsymbol{S}^d_i - z^s_i \boldsymbol{Y}^s_i |V_i|^2 = \sum_{(i,j) \in E \cup E^R} S_{ij} \ \ \forall i \in N$$

$$S_{ij} = \left(\boldsymbol{Y}^*_{ij} - \boldsymbol{i} \frac{\boldsymbol{b}^{\boldsymbol{c}}_{ij}}{2} \right) \frac{|V_i|^2}{|\boldsymbol{T}_{ij}|^2} - \boldsymbol{Y}^*_{ij} \frac{V_i V^*_j}{\boldsymbol{T}_{ij}} \ \ (i,j) \in E$$

$$S_{ji} = \left(\boldsymbol{Y}^*_{ij} - \boldsymbol{i} \frac{\boldsymbol{b}^{\boldsymbol{c}}_{ij}}{2} \right) |V_j|^2 - \boldsymbol{Y}^*_{ij} \frac{V^*_i V_j}{\boldsymbol{T}^*_{ij}} \ \ (i,j) \in E$$

$$|S_{ij}| \le \boldsymbol{s}^{\boldsymbol{u}}_{ij} \ \ \forall (i,j) \in E \cup E^R$$

$$-\boldsymbol{\theta}^{\boldsymbol{\Delta}}_{ij} \le \angle (V_i V^*_j) \le \boldsymbol{\theta}^{\boldsymbol{\Delta}}_{ij} \ \ \forall (i,j) \in E$$

$$\sum_{i \in N} \boldsymbol{M}^v z^v_i + \boldsymbol{M}^g z^g_i + \boldsymbol{M}^s z^s_i + |\Re(\boldsymbol{S}^d_i)| z^d_i$$

Multi-Objective
Implementation trick

Key Questions

- Do non-convex solvers “just work” in practice (like OPF?)
  - If so, is there a large optimality gap?
- How much do we loose by convexifing?
  - AC to SOC
  - Discrete to Continuous
  - Which problem features are important?
- Does AC-MLS still seem “maddeningly difficult”?

- 4 Variants
  - AC-MLS (MINLP)
  - AC-MLS-C (NLP)
  - SOC-MLS (MISOCP)
  - SOC-MLS-C (SOCP)

Experimental Evaluation

Experiment Design and Test Cases

- PGLib Test Cases Ranging from 73 to 6468 buses
  - Scaling properties
  - Test realistic sizes
- 1000 random N-30% damage scenarios (branch only)
  - Was the hardest case to find feasible solutions in the 2011 study
- Solvers: Bonmin, Pajarito (ipopt+gurobi), Ipopt (HSL ma27)

Test Case	$\|N\|$	$\|E\|$	k	Scenarios
IEEE RTS 96	73	120	36	1000
PSERC 240	240	448	134	1000
PEGASE 1354	1354	1991	597	1000
RTE 1888	1888	2531	759	1000
Polish 2383wp	2383	2896	869	1000
Polish 3120sp	3120	3693	1108	1000
RTE 6468	6468	9000	2700	1000

Convergence Results

Status	Solver Status Breakdown AC-MLS	AC-MLS-C	SOC-MLS	SOC-MLS-C	Average Runtime (seconds) AC-MLS	AC-MLS-C	SOC-MLS	SOC-MLS-C
IEEE RTS 96 (n=1000)								
converged	98.60%	94.40%	100.00%	100.00%	16.18	0.14	0.50	0.07
time limit	1.40%	–	–	–	1526.11	–	–	–
error	–	5.60%	–	–	–	150.00	–	–
PSERC 240 (n=1000)								
converged	71.30%	98.70%	100.00%	100.00%	18.46	1.54	4.10	0.32
time limit	3.70%	0.70%	–	–	1614.58	69.82	–	–
error	25.00%	0.60%	–	–	1500.00	20.63	–	–
PEGASE 1354 (n=1000)								
converged	94.10%	100.00%	100.00%	100.00%	221.36	5.46	32.97	2.26
time limit	1.70%	–	–	–	1598.14	–	–	–
error	4.20%	–	–	–	1500.00	–	–	–
RTE 1888 (n=1000)								
converged	84.30%	88.40%	100.00%	100.00%	99.38	10.53	67.21	2.53
time limit	0.10%	11.50%	–	–	1655.67	151.00	–	–
error	15.60%	0.10%	–	–	1500.00	130.41	–	–
Polish 2383wp (n=1000)								
converged	86.00%	100.00%	99.80%	100.00%	121.58	7.64	38.43	3.04
time limit	1.30%	–	–	–	1639.76	–	–	–
error	12.70%	–	0.20%	–	1500.00	–	990.95	–
Polish 3120sp (n=1000)								
converged	15.70%	99.90%	99.80%	100.00%	659.79	9.49	67.67	4.03
time limit	74.00%	0.10%	–	–	1531.44	150.75	–	–
error	10.30%	–	0.20%	–	1500.00	–	633.83	–
RTE 6468 (n=1000)								
converged	14.50%	35.60%	99.60%	100.00%	865.42	57.17	648.47	12.34
time limit	3.10%	58.70%	–	–	1608.88	152.38	–	–
error	82.40%	5.70%	0.40%	–	1500.00	92.89	1500.00	–

SOC-MLS-C rocks!

For small networks, all formulations work

For large networks, AC feasibility is a significant issue on large cases

SOC Relaxation is fast and reliable

What about Opt. Gaps? SOC-C ok?

Optimality Gaps

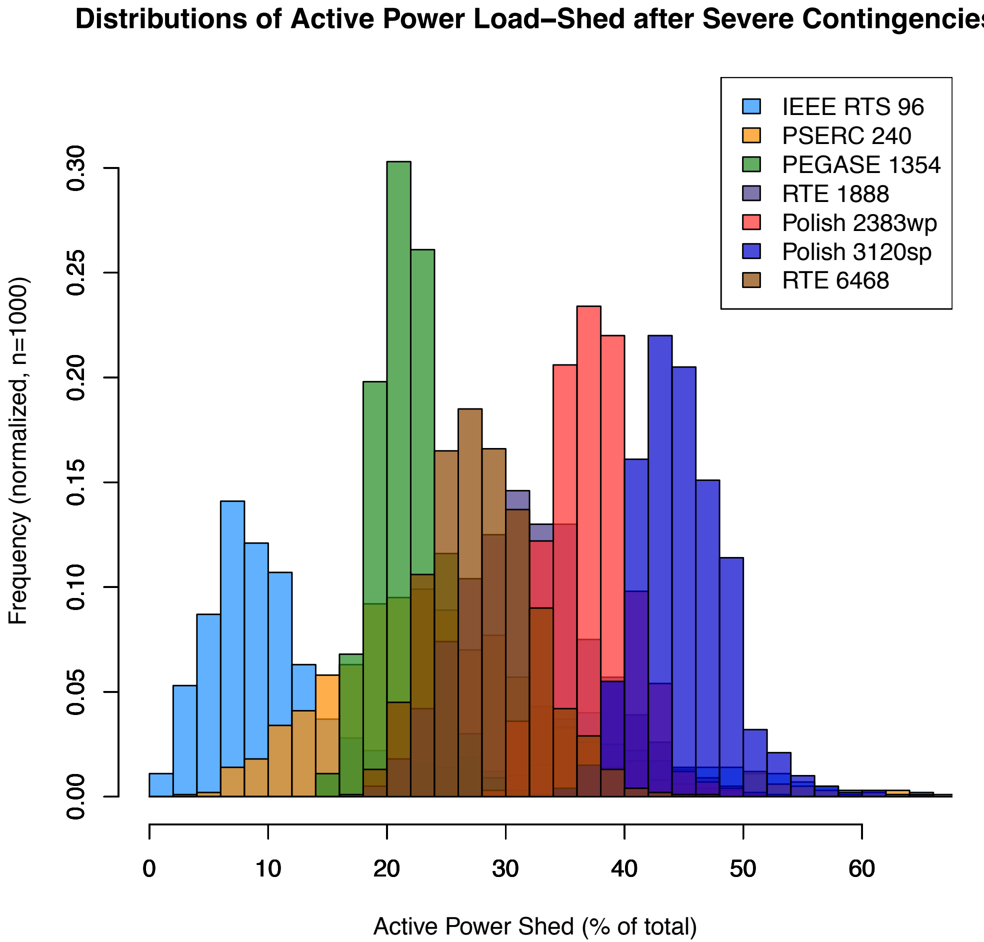
Case	Obj. Val. AC-MLS	Optimality Gap (%) AC-MLS-C	SOC-MLS	SOC-MLS-C
IEEE RTS 96 (n=914)	2.463e+04	0.0000%	0.0044%	0.0044%
PSERC 240 (n=707)	1.945e+06	-0.0049%	0.0010%	0.0010%
PEGASE 1354 (n=927)	2.001e+06	0.0022%	0.0024%	0.0037%
RTE 1888 (n=728)	1.010e+06	-0.0267%	0.0006%	0.0006%
Polish 2383wp (n=859)	5.759e+05	0.0064%	0.0062%	0.0077%
Polish 3120sp (n=155)	1.078e+06	0.0079%	0.0138%	0.0186%
RTE 6468 (n=54)	9.496e+06	-0.0151%	0.0003%	0.0003%

Those are some small gaps!

Proof-of-Concept Load Shedding Study (SOC-MLS-C)

Mean / Variance of Large Scale Branch Damage (i.e. 30%)

Reminiscent of Hurricane threat



Great Variety in Distributions

What network features lead to N-30% variability?

Conclusions

Conclusions

- SOC-MLS-C is surprisingly good!

- Convex Relaxation + Random Test Generation
  - Proving solution in-existence of AC problems is hard
  - Worked well for developing a seemingly feasible AC-MLS formulation

- Future Work
  - Use SOC-MLS-C solutions to build AC-MLS feasible solutions

- Special thanks to co-authors (e.g. Russell, Byron, Kaarthik, Scott)

Thanks!

https://arxiv.org/abs/1710.07861